8.3

The Hyperbola

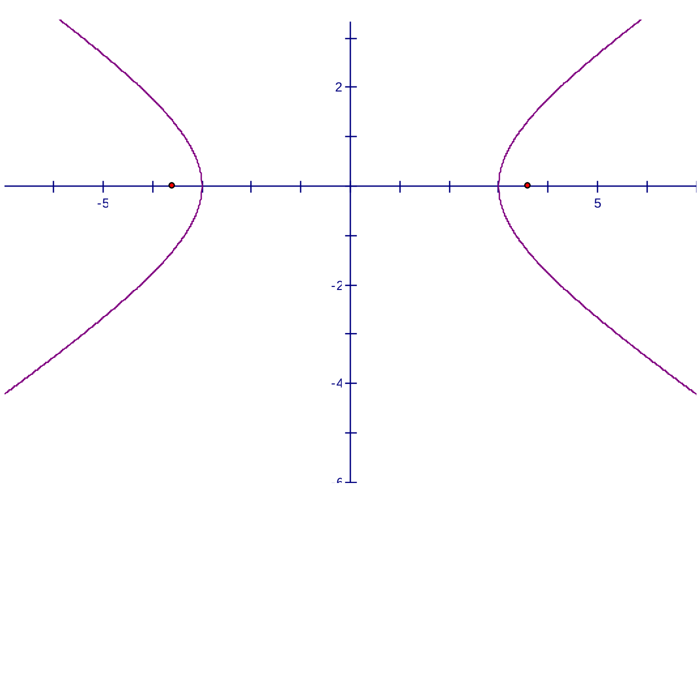
A **hyperbola is** the collection of all points in the plane the difference of whose distances from two fixed points, called the **foci**, is a constant.

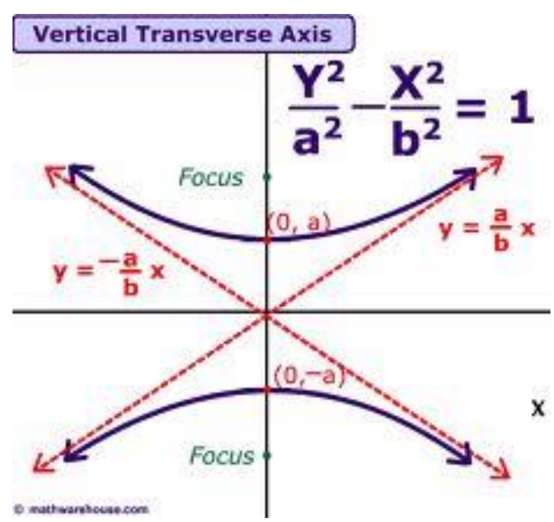
The line containing the foci is called the **transverse axis.** The midpoint of the line segment joining the foci is the **center** of the hyperbola.

The line through the center and perpendicular to the transverse axis is the **conjugate axis**.

The hyperbola consists of two separate curves, called **branches,** that are symmetric with respect to the transverse axis, conjugate axis, and the center.

The two points or intersection of the hyperbola and the transverse axis are the **vertices**, **V_1 and V_2**, of the hyperbola.





Equation of a Hyperbola

Center at (0, 0); Foci at (±c, 0); Vertices at (±*a*, 0);

Transverse Axis along the x-Axis

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad where \;\; b^2 = c^2 - a^2$$

Find an equation of the hyperbola with center (0, 0), one focus at (3, 0) and one vertex at (-2, 0).

We need to know *a* and *b* to write the equation. Given one vertex at (-2, 0) we know *a* = 2. Given one focus at (3, 0) we know c = 3. So,

$$b = \sqrt{c^2 - a^2} = \sqrt{9-4} = \sqrt{5}.$$

Therefore the equation is: $\frac{x^2}{4} - \frac{y^2}{5} = 1$

Discuss the equation $\frac{x^2}{16} - \frac{y^2}{4} = 1$

Equation of a Hyperbola

Center at (0, 0); Foci at (0, $\pm$c); Vertices at (0, $\pm$*a*);

Transverse Axis along the *y*-Axis

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad where \;\; b^2 = c^2 - a^2$$

Discuss the equation: $y^2 - 4x^2 = 4$

Find an equation of the hyperbola having one vertex at

(0, 2) and foci at (0,-3) and (0, 3)

Asymptotes of a Hyperbola

The hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ has the two oblique asymptotes

$y=\dfrac{b}{a}x \quad and \quad y=-\dfrac{b}{a}x$

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Hyperbolas with Center at (h, k) and Transverse Axis Parallel to a Coordinate Axis

Center: (h, k) Transverse Axis: parallel to x-axis

Foci: $(h \pm c, k)$ Vertices: $(h \pm a, k)$

Equation: $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1, \quad b^2=c^2-a^2$

Asymptotes: $y-k=\pm\dfrac{b}{a}(x-h)$

Center: (h, k) Transverse Axis: parallel to y-axis

Foci: $(h, k \pm c)$ Vertices: $(h, k \pm a)$

Equation: $\dfrac{(y-k)^2}{a^2}-\dfrac{(x-h)^2}{b^2}=1, \quad b^2=c^2-a^2$

Asymptotes: $y-k=\pm\dfrac{a}{b}(x-h)$

The eccentricity of a hyperbola is $e = \frac{c}{a} = \frac{\sqrt{a^2 + b^2}}{a}$ where *a* is the semitraverse axis and *b* is the semiconjugate axis.